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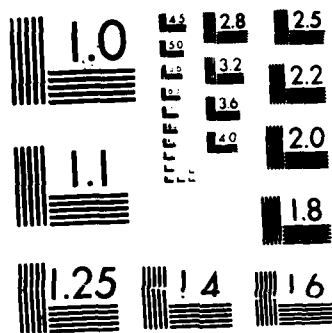
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A CRITIQUE OF
CONSTRAINED FACET ANALYSIS

TECHNICAL REPORT

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April 1987

AU-AFIT-LSY-87-1

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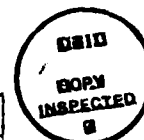
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ABSTRACT

This critique discusses the development, limitations, and advantages of an efficiency measurement methodology called Constrained Facet Analysis (CFA). CFA is a variant of Data Envelopment Analysis (DEA) which evolved in response to limitations in the application of DEA experienced by the Educational Productivity Council at The University of Texas at Austin. However, CFA has its own inherent limitations which analysts need to be aware of when applying CFA. It is concluded that application of CFA should be withheld until CFA is further developed and tested.

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I. INTRODUCTION

In the Summer 1984 issue of Air Force Journal of Logistics T. Clark, A. Bessent, E.W. Bessent, and J. Elam essayed what they termed as a "new method of computing efficiency", Constrained Facet Analysis (CFA). In that article they focused on the military's need for integrative models of efficiency and capability and illustrated how CFA might fill that void. The purpose of this critique is to elaborate on some of the assumptions, characteristics, and capabilities of CFA.

II. EVOLUTION OF CFA

Constrained Facet Analysis evolved from Data Envelopment Analysis (DEA), an efficiency measurement methodology developed by A. Charnes, W.W. Cooper, and E. Rhodes (CCR) (1978 and 1979) and extended and tested by others¹, as a result of the experience of the Educational Productivity Council (EPC). The problem experienced by the EPC was the perception that DEA significantly overestimates organizational efficiency if significant amounts of slack are present in the DEA solution.² Clark et. al. (1984) also note that DEA is limited in its ability to provide planning information, i.e., rates of substitution and marginal productivities.³

To understand these problems we need to review the DEA model and its efficiency measure. We will elaborate on the DEA efficiency measure by referring to the following CCR linear-programming formulation of the DEA model.

$$\begin{aligned}
 &\text{Minimize : } h_0 = \theta - \epsilon \left(\sum_{r=1}^s s_r^+ + \sum_{i=1}^m s_i^- \right) \\
 (1) \quad &\text{Subject to: } \sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{r0}; \quad r=1, \dots, s \\
 &\quad \quad \quad \sum_{j=1}^n x_{ij} \lambda_j + s_i^- - \theta x_{i0} = 0; \quad i=1, \dots, m \\
 &\quad \quad \quad \lambda_j, s_i^-, s_r^+ \geq \epsilon > 0 \\
 &\quad \quad \quad \theta \text{ unrestricted in sign}
 \end{aligned}$$

where:

θ = An intensity value or multiplier of the observed input values.

s_r^+ = Output slack for output "r".

s_i^- = Input slack for input "i".

$\epsilon > 0$ = A small positive valued non-Archimedean constant.

y_{r0} = The known amount of output "r" produced by the unit being evaluated during the period of observation.

x_{i0} = The known amount of input "i" used by the unit being evaluated during the period of observation.

y_{rj} = The known amount of output "r" produced by unit "j" during the period of observation.

x_{ij} = The known amount of input "i" used by unit "j" during the period of observation.

λ_j = Multiplier determined by the model which indicates the evaluated unit's comparison set.

In transforming DEA from its original non-linear fractional form into the more easily used and beneficial linear programming form shown above, Charnes, Cooper, and Rhodes had to apply the concept of non-Archimedean values. To keep this critique from becoming overly technical we will not detail the transformation nor the non-Archimedean concept and the solving of the model.⁴ Let it suffice to say that this transformation results in DEA reporting efficiency measurement through two factors which are reduced to a single scalar measure of efficiency via the non-Archimedean variable, ϵ .

The two efficiency factors resulting from solving the DEA model are designated in the objective function of (1). Part one of the measurement is reflected in the θ^* value which indicates technical and scale inefficiencies.⁵ A second part of the efficiency measurement is evidenced in the slack values, s_r^+ and s_i^- , which indicate mix inefficiencies. The impact of these slack variables and the inefficiencies they

represent are eliminated from the h_0^* computed by the DEA model via the non-Archimedean constant, ϵ . The non-Archimedean constant is an infinitesimally small number, and hence, from the objective function, we have:

$$(2) \quad \left(\sum_{r=1}^s s_r^+ + \sum_{i=1}^m s_i^- \right) \rightarrow 0$$

and thus: $h_0^* = \theta^*$. Consequently, the h_0^* value reflects only part of the inefficiencies identified by the DEA model since the slack variables are not reported as part of the h_0^* value.

Retaining the concept of non-Archimedean values for identifying slack values as DEA does can be awkward, not only for computation, but also for uses such as comparing overall organizational efficiency and studying statistical behavior of efficiencies where restrictions to a single real valued measure of overall efficiency is required. The appearance of positive slack values in a solution also inhibits the computation of marginal rates of substitution and productivity.

Hence, the problem experienced by the EPC and others was twofold. First, how does an analyst combine the two factors into a single overall measure of efficiency so as to facilitate comparing the operational efficiency of organizations. Second, how does an analyst compute planning factors such as marginal rates of substitution and productivity for those variables in which slack variables appear.

Clark's variant of DEA, Constrained Facet Analysis, is an attempt to rectify the shortcoming of having slack values in the DEA efficiency solution. CFA provides both an upper and lower bound measures of efficiency. The upper bound is actually the solution to the DEA model. Therefore, this critique refers mainly to CFA's lower bound measure of efficiency since this is unique to CFA and is CFA's contribution to this area of research.

III. THEORETICAL DEVELOPMENT OF CFA

The approach used by Clark, et. al. is best described in connection with Figure 1.⁶ For simplicity we restrict ourselves to the case of one output (produced at unit level) and two inputs, x_1 and x_2 . Hence any inefficiencies are in the input amounts utilized. The x_1 and x_2 coordinates of points (= organizations) A, B, C, D, and E represent observed inputs used to produce the one unit of output attained by each of the five units associated with these points. The solid line connecting points A, B, and C represents a section of the unit isoquant, i.e., all organizations have produced one unit of a single output, and is an "efficiency frontier".

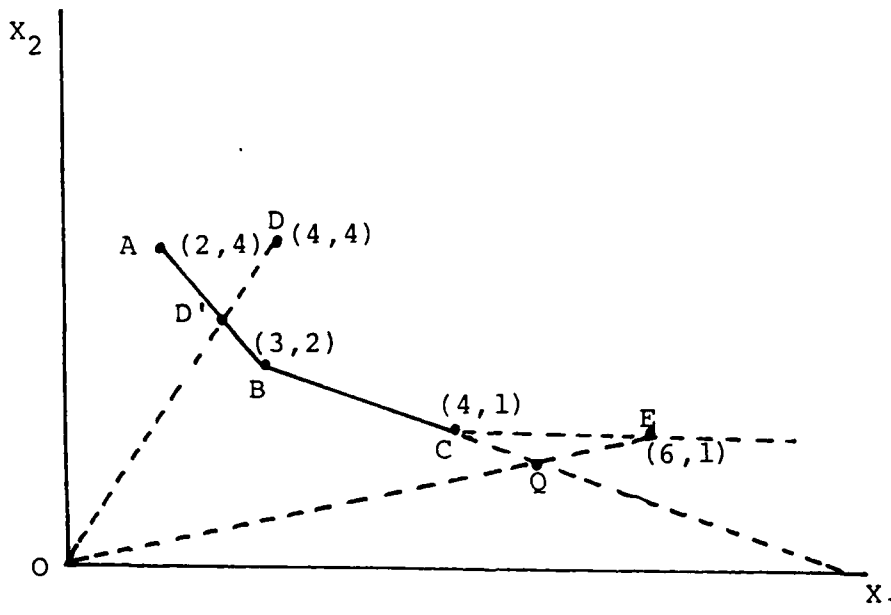


FIGURE 1: Efficiency In The Single Output - Two Input Case

Points A, B, and C are on the efficiency frontier and would be rated 100% efficient by CFA. Point D is a "fully

enveloped"⁷ inefficient unit. It is off of the efficiency frontier and its upper and lower bound measures of inefficiency are the same (i.e., CFA = DEA) and equal to the ratio OD'/OD .

Now consider the point E exhibited in Figure 1. Point E is an "outlier"⁸ and "not fully enveloped". For E to obtain its unit (rate) of output, it used input $x_2 = 1$, the same as C, but it also used input $x_1 = 6$ which exceeds the value used by C for this input by 2 units or 50%. The approach suggested by Clark et. al. projects the frontier from the edge connecting B and C until it meets the axis. The ratio OQ/OE is then used as the measure of efficiency.

Staying with the simple situation presented in Figure 1, we appeal to the concept of Pareto optimality.⁹ Note that movement from E to C may be affected in a way that reduces x_1 from $x_1 = 6$ to $x_1 = 4$ without requiring any increase in $x_2 = 1$. Since, by definition, we remain on the same isoquant the output level also is not disturbed. Consequently, E is not Pareto optimal, i.e., a worsening of x_2 is not required in exchange for a betterment of x_1 .

The point at C, however, is Pareto optimal and so is every point on the frontier connecting B and C. Movement along this frontier can be effected only if at least one input is increased in order to achieve a diminution of the other input, while holding output fixed. The extrapolation from C to Q represents an extrapolation of this same Pareto optimality property.

Returning to the use of OQ/OE as a measure of efficiency we can see, by reference to the diagram, that the movement from E to O involves a reduction in both x_1 and x_2 . Furthermore, the movement from E to Q fails to reduce x_1 to the value it would achieve at C in exchange for a reduction in x_2 which would not have been achieved at C. Note also, that we are now outside the empirical production possibility set defined by the original observations (range of observed data).

Justification for this approach is required, as is always the case for movement along any Pareto efficient frontier, since a trade-off between two resources is implicitly required. In other words, the CFA approach implies a rate of exchange for substitution between x_1 and x_2 which is not reflected in the observed data. From a management standpoint, the manager of unit E is being asked to reduce both inputs and may well demand to be shown the evidence that this can be done without lowering output. This type of evidence is not available under CFA for the imputed lowering of both inputs. Hence, the manager may challenge the CFA results as being unattainable since there is no evidence that the efficient input/output levels determined by CFA are attainable.

Note at this point an important assumption of CFA. CFA's lower bound measure of efficiency for "outlier" units is based on an efficiency frontier which is extended outside the range of observed data. Putting this assumption another way, we can say that CFA's lower bound efficiency frontier is computed

through extrapolation. Perhaps the significance of this assumption and the dangers involved can best be illustrated with an analogy to regression analysis.

A basic tenet of regression analysis is that an estimating function is established on the basis of a particular set of observations. Consequently, care must be taken in predicting (extrapolating) values of the dependent variable from values of an independent variable which is outside the range of observed data. Extrapolating beyond the range of observed data assumes that the predetermined functional relationship continues. Without any additional information, this assumption is unsupported since we do not know whether the functional form is valid outside the range of observed data.

The hazards of extrapolation are indicated by the confidence interval around a prediction. The boundaries of this interval become wider as the estimated value moves further from the intersection of the means and it becomes more and more difficult to have any confidence in the accuracy of the estimate. This same generalization could be applied to CFA's lower bound efficiency measure -- it becomes more and more difficult to be confident in the accuracy of the estimate the further we move outside the range of data. However, unlike regression analysis, there is no way of constructing a confidence interval around the CFA estimate to indicate this hazard.

IV. TESTING CFA

Bowlin (1984) exhibited the kinds of situations that may develop by testing CFA against a known technology. This was done by applying CFA to a data set developed by H.D. Sherman (1981) which contained 15 hypothetical decision making units (DMU) with known efficiencies and inefficiencies.¹⁰ There were seven efficient DMUs and eight DMUs with inefficiencies. In addition, relations within the data such as complementarity and substitution were taken into account in developing the data base. Thus, this data set is well suited for testing the ability of different methodologies to identify sources and amounts of inefficiencies in an organization's operations and correspondingly, estimate efficient input and output levels.

The results of this test are displayed in Table 1. The efficient hypothetical DMUs, H1 to H7, are omitted since CFA correctly identified these units as being efficient and there is no issue. However, for the inefficient DMUs, H8 to H15, the CFA estimates of the known true efficient input values were often very wide of the mark and generally very erratic with no clear signal of possible trouble being apparent. In some cases CFA estimates were significantly greater than the true efficient value while with other cases the CFA estimates were significantly under the true value. Note, in fact, that both circumstances can occur for the same DMU, i.e., the efficiency estimates can be both under and over the true

efficiency values, as can be seen from H11 where CFA greatly underestimates the efficient level for input FTE and overestimates the efficient input level for \$\$.

Table 1

Constrained Facet Analysis Estimates

(1) DMU	(2) Observed Value	(3) Input Label	(4) CFA Est.	(5) True Eff Value	(6) % Diff
H8	25.0	FTE	25.0	25.0	0.0
	49,475	BD	49,475	41,053	20.5+
	140,000	\$\$	140,000	140,000	0.0
H9	24.5	FTD	17.1	24.5	30.2-
	43,160	BD	30,081	43,158	30.3-
	165,000	\$\$	140,000	140,000	0.0
H10	77.0	FTE	77.0	53.0	45.0+
	92,630	BD	92,630	92,630	0.0
	340,000	\$\$	340,000	280,000	21.4+
H11	44.5	FTE	26.5	36.5	27.4-
	65,260	BD	65,259	65,263	0.0
	265,000	\$\$	264,997	200,000	32.5+
H12	30.0	FTE	30.0	30.0	0.0
	60,000	BD	59,940	50,526	18.6+
	170,000	\$\$	169,830	170,000	.1-
H13	43.5	FTE	43.5	43.5	0.0
	81,110	BD	81,110	76,842	5.6+
	245,000	\$\$	245,000	240,000	2.1+
H14	30.0	FTE	30.0	30.0	0.0
	60,000	BD	59,940	50,526	18.6+
	170,000	\$\$	169,830	170,000	.1-
H15	26.5	FTE	23.5	23.5	0.0
	47,370	BD	42,007	41,053	2.3+
	160,000	\$\$	130,000	130,000	0.0

Key: FTE - Full time equivalent of labor
 BD - Available bed days
 \$\$ - Supply dollars

Note: Computations were done using computer codes obtained from the Educational Productivity Council at The University of Texas at Austin.

V. SUMMARY AND CONCLUSION

In summary, Clark et. al. propose Constrained Facet Analysis as a method of measuring efficiency of the Air Force and other non-profit organizations. Their approach provides an upper bound measure of efficiency which is the DEA measure and a lower bound measure of efficiency. Our analysis concentrated on the lower bound measure since this is CFA's contribution to efficiency measurement research.

CFA does address some of the inherent limitations in DEA caused by using the non-Archimedean concept. However, CFA has some critical limitations of its own. Its lower bound is based on an efficiency frontier that might be outside the range of observed data. Consequently, the farther the estimated efficiency frontier extends from the observed data points, the less confidence we might have in the estimated efficient input and output levels actually being attainable. Also, tests of CFA with data from a known technology showed that this approach can lead to estimates which are erratic and significantly different from those that are the true efficiency values, and they may not be attainable. These findings may explain Clark et. al.'s (1984) results for Wing K which, as they report, are unattainable.

At the current state of development, it appears that CFA's lower bound efficiency measure has limited application. Until some of the questions raised in this comment are

resolved, analysts may want to consider using Data Envelopment Analysis (DEA) as an alternative to CFA for estimating efficient input and output levels. CFA is a variant of DEA and all the characteristics and advantages, e.g., not requiring inputs and outputs to have common units of measurement, attributed to the CFA model by Clark et. al. have their origin in the DEA model. Tests by Bowlin et. al. (1985) show that DEA's efficiency estimates are accurate while remaining within the relevant range of the data. Finally, there are several extensions of DEA such as distinguishing between technical and scale efficiencies and incorporating categorical and nondiscretionary variables into the analysis that make DEA an attractive alternative for measuring efficiency.

NOTES

1. See, for example, Banker, Charnes, Cooper (1984); Banker (1984), and Banker and Morey (1986a and 1986b) for extensions of DEA and Bowlin (1987) and Bowlin, Charnes, Cooper, and Sherman (1985) for tests of DEA.
2. See Clark (1983) pp. 28 & 144.
3. Although Clark et. al. do not state what these limitations are, they can be found in Clark (1983).
4. Charnes et. al. (1978 and 1979) show the transformation from the original non-linear fractional programming problem to the linear programming problem via the theory of linear fractional programming developed by Charnes and Cooper (1962).
5. See Banker (1984).
6. This figure was adapted from Clark (1983) p. 104.
7. Fully enveloped refers to the situation where an inefficient unit is fully explained by a convex combination of other units on the efficiency frontier and there are no positive slack values. Point D is fully explained by a convex combination of points A and B.
8. Using Clark's terminology, an "outlier" is a unit whose input and output measures are extreme when compared to the ranges of values for frontier units. It is a unit that is not fully explained by a convex combination of frontier units and will have some positive slack values.
9. Pareto optimality is the state obtained in an economy when one input cannot be improved (decreased) without causing at least one other input to be worse off (increase). See Cooper and Ijiri (1983) p. 373.
10. See also Appendix A in Bowlin (1984) and Bowlin et. al. (1985) for a description of this data base.

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